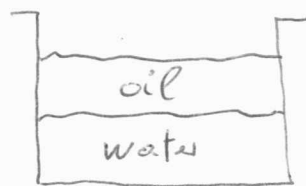# The hydrophobic interaction :

non polar ("hydrophobic") substances have a low
solubility in water, e.g.:

solubility constant K
= solubility in mole fraction

i.e. you consider the "reaction" $Oil \longrightarrow Oil_{aq.}$

and $\Delta G^{\circ} = - RT \ln X$

$R = N_0 k$, X molar fraction
of oil in water at eq.
(molar fraction of oil in oil $= 1$!)

$K = X$ so $\Delta G^{\circ} = - RT \ln K$.   This is how
you measure ΔG°.

ΔG° is the change in Gibbs pot. if you transfer oil
into water : $\Delta G(\ell \rightarrow w) = G(w) - G(\ell)$
ℓ : "liquid" , w : "water"

In gen. for a chemical reaction $A + B \rightleftharpoons C$
using $\mu = \mu^{\circ} + kT \ln X$, μ° chem. pot. of
the pure component

writing for one mole :

$$G_A = G_A^\circ + RT \ln X_A \quad , \quad etc.$$

ΔG for the reaction is $\quad \Delta G = G_C - (G_A + G_B)$

(i.e. you start with 1 mole of A , 1 mole of B, and

     end up with 1 mole of C)

$$\Rightarrow \quad \Delta G = \underbrace{G_C^\circ - (G_A^\circ + G_B^\circ)}_{\Delta G^\circ} + RT \ln \frac{X_C}{X_A X_B}$$

ΔG° molar Gibbs pot. difference between pure

     and products and pure reactants .

At eq. $\quad \Delta G = 0 \quad \Rightarrow \quad \Delta G^\circ = - RT \ln K$

     with $\quad K = \dfrac{X_C}{X_A X_B} \qquad \left[\begin{array}{l} i.e. \; at \; eq. \\ \mu_A + \mu_B = \mu_C \end{array} \right]$

$\left[\text{Note :} \quad \text{if you use concentrations} , \quad X_C = \dfrac{M_C}{M_A + M_B + M_C} \right.$

etc. $\quad \Rightarrow \quad \dfrac{X_C}{X_A X_B} = \dfrac{M_C}{M_A M_B} (M_A + M_B + M_C) = \dfrac{M_C / V}{\dfrac{M_A}{V} \dfrac{M_B}{V}} \dfrac{M_{tot.}}{V}$

$= \dfrac{[C]}{[A][B]} \; C_{tot.} \qquad \Big]$

$\longrightarrow$

Or also:
$$A + B \rightleftharpoons C$$

$$\mu = \mu_0 + kT \ln \frac{C}{C_0}$$

(C_0 ref. conc. at which $\mu = \mu_0$)

eq.: $\mu_A + \mu_B = \mu_C$

$$\Rightarrow kT \ln \frac{C_A C_B}{C_c} \frac{1}{C_0} = \mu_0^{AB} - (\mu_0^A + \mu_0^B) = \Delta\mu_0$$

$$\Rightarrow \frac{C_A C_B}{C_c} = C_0 e^{\Delta\mu_0 / kT} = C_0 \, K(T)$$

or : $\frac{[A][B]_{eq.}}{[C]_{eq.}} = (1M) e^{\Delta G^\circ / RT}$

ΔG° free energy diff. if all reactants are present at $1M$ conc.

In non-eq., what is ΔG?

$$\Delta\mu = \mu_C - (\mu_A + \mu_B) = - kT \ln \frac{C_A C_B}{C_c} \frac{1}{C_0} + \Delta\mu^\circ$$

$$\Rightarrow \Delta G = - kT \ln \frac{[A][B]}{[C]} + \Delta G^\circ$$

So low solubility (hydrophobic) means $K \ll 1$

$\rightarrow$ $\Delta G > 0$ and large $\left(\Delta G(\ell \rightarrow w) \right)$.

On the other hand, these substances also have

$\Delta H(\ell \rightarrow w) \simeq 0$ (small) — You can ~~never~~ measure ΔH by calorimetry —

$\Delta G = \Delta H - T \Delta S$ so for these substances

$\Delta S(\ell \rightarrow w)$ is large and negative ~~an~~ (room temp.).

E.g.: benzene (room temp.)

K (mole fraction)	ΔG (kJ/mole)	ΔH	$T\Delta S$
4×10^{-4}	19	2	-17

and remember, $20 \text{ kJ/mole} \simeq 8 \text{ } kT/\text{molecule}$

This large entropy decrease is due to a partial ordering of the water molecules around the hydrophobic molecules; several water molecules must be involved ~~(since ΔS ~~~~~~)~~ (since $\Delta S \sim 7 \div 8 \text{ } k$!). Why do the water molecules order? They have less choices for hydrogen bonding if they are close to a hydrophobic surface (i.e.
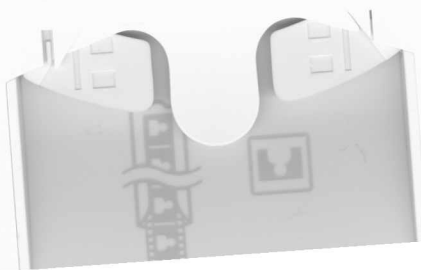
the hydrogen bonding network is more ordered close
to a hydrophobic surface) .

So the hydrophobic interaction is really the water trying
to expel the hydrophobic molecules, which leads to a
tendency to aggregate (i.e. an effective attractive interaction).

   Range ?

The same effect also accounts for the high surface tension
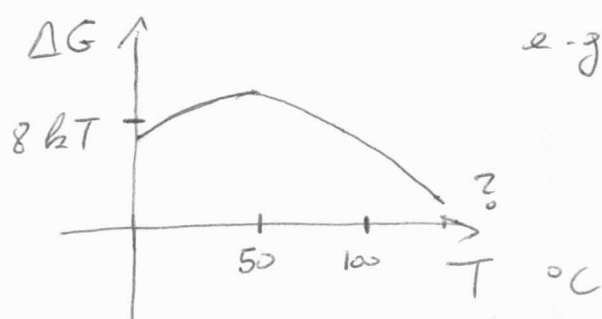of water   (which you can reduce with a surfactant :

water
)
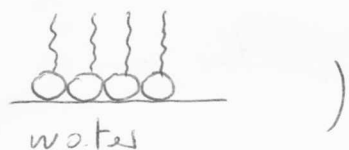
Hydrophilic molecules :   − charged (ion − dipole interact.)
                             − polar (dipole − dipole −) .

<u>Temperature dependence</u> of the hydrophobic int. :
    interaction increases with T in some range
   $(-T \Delta S)$, but $\longrightarrow 0$ at high enough T
   $(110 - 140 \, ^\circ C)$ because then you have destroyed the
   ~~ordered it~~ extra ordered structure   around the hydrophobic mol.

e.g. benzene in water
   (relative to benzene in benzene)
   for $T \sim 140^\circ C$, $\Delta G \approx 0$?

<u>Important</u> :   biomolecular interactions

depend on temp. !!

I.e. the actual interaction strength changes with T

So e.g. when a protein unfolds, it's not just
because of the $T\Delta S$ term in $F = E - TS$
with ΔS ~ temp. indep. ; instead, ΔS itself
(and ΔE) have strong temp. dependencies .
See e.g. model of "cold unfolding" later on .

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Hydrogen bonds.   E.g. in water

main feature is :   directionality

approx. as interaction energy :

$$E_{HB} \sim D_0 \left[5\left(\frac{R_0}{R}\right)^{12} - 6\left(\frac{R_0}{R}\right)^{10} \right] \cos^4 \theta$$

$D_0 \sim 8 \, kcal/mole$
$\simeq 13 \, kT$

θ deviation from "ideal" alignment.

Directionality $\Rightarrow$ secondary structure of proteins
(α-Helics, β-sheets) ; DNA double helix, etc.

But remember : in the case of e.g. DNA , you
have to compare hydrogen bonding between AT say
to hydrogen bonding with the water $\Rightarrow$ stability is
only $\sim$ 1 kT per hydrogen bond.
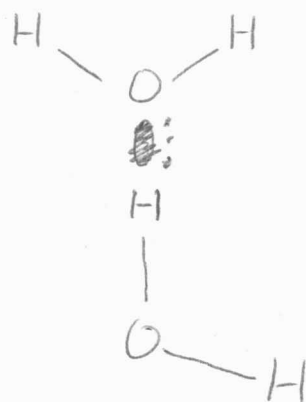
Fundamental interactions

Van der Waals forces:

consider two atoms at a distance $\vec{r}$

a quantum fluct. in the first results in a dipole moment P_1

$\longrightarrow$ field $E = \dfrac{P_1}{r^3}$; the second atom has polarisability α

$\longrightarrow$ induced dipole $P_2 = \alpha E = \alpha \dfrac{P_1}{r^3}$; the field of this dipole at position 1 is $E_2 = \dfrac{P_2}{r^3} = \alpha \dfrac{P_1}{r^6}$

$\longrightarrow$ energy $\phi = - P_1 E_2 = - \alpha \dfrac{P_1^2}{r^6}$

Bohr atom:

$$P_1 = e a_0 \quad , \quad a_0 = \frac{e^2}{2 h\nu} \quad \text{Bohr radius}$$

$h\nu$ = ionisation energy ;

polarisability: $\alpha \simeq a_0^3$

$\left[\text{note}: \quad [\alpha] = \ell^3 \right]$

e.g. for a dielectric sphere, $\alpha = \dfrac{\varepsilon - 1}{\varepsilon + 2} a^3$

$\Rightarrow \quad \phi \sim - \alpha^2 \dfrac{e^2}{a_0} \dfrac{1}{r^6} \sim - \alpha^2 \dfrac{h\nu}{r^6}$

VdW interaction between two atoms.

The London eq. (from q.m. perturbation theory)

is the same with a factor 3/4 :

$$\phi = -\frac{3}{4} \alpha^2 \frac{h\nu}{r^6}$$

According to our estimate, $\alpha \approx a_0^3 \approx 10^{-24} \, cm^3$

another estimate is from a harmonic oscillator

model : $m \omega^2 \vec{x} = e \vec{E} \implies \alpha = \frac{e^2}{m \omega^2}$

with $\lambda \sim 300 \, m\mu$ (near UV) $\implies \omega \sim 6 \times 10^{15} \, H_z$

$\implies \alpha \sim 6 \times 10^{-24} \, cm^3$

Strength of the VdW forces =   consider two atoms at
                                                    contact : $r \approx 0.3 \, nm$

$\alpha \approx 10^{-24} \, cm^3$, $h\nu \approx 10 \, eV = 400 \, kT$

$\implies \phi_{VdW} \approx 10^{-48} \frac{400}{650 \times 10^{-48}} kT \sim 1 \, kT$

These are the forces responsible for the condensation
(liquid & solid phases) of non-ionic, non polar substances
e.g. He, Ne, Ar, CH_4 etc.

The London eq. predicts the right cohesive energy for
VdW solids , e.g. Ar :   closed packed solid $\Rightarrow$

12 nearest neighbors $\longrightarrow$ 6 full bonds per atom

$\longrightarrow$ molar energy $= N_0 \times 6 \times \frac{3}{4} \alpha^2 \frac{h\nu}{r^6}$

Taking into account all neighbors, $6 \rightarrow 7.2 \ldots$
 and using the interatomic distance for solid Ar
 $r = 3.76 \, \overset{\circ}{A}$ etc.   you get 7.7 kJ/mole

$\left(10 \text{ kJ/mole} \simeq 4 \, kT / \text{molecule} \right)$ _

What you measure is the latent heat of melting
+ latent heat of vaporization, which is also 7.7 kJ/mole !

In gen. there are other contributions to the VdW interaction
(e.g. if the molecules have a permanent dipole moment,
 like H_2O), but they also go as $\sim r^{-6}$ _

To calculate the coeff. from the microscopics is
a bit messy _ Here we just show that the interaction
is not necessarily always attractive, because =

  For macroscopic bodies , write $V = -\frac{A}{n_1 n_2} \frac{1}{r^6}$

  across vacuum, $A \simeq 10^{-12}$ ergs

the London eq. for two dissimilar atoms is

$$\phi = -\frac{3}{2}\frac{\alpha_1 \alpha_2}{r^6}\frac{h\nu_1\nu_2}{(\nu_1+\nu_2)}$$

in a medium, e.g. for a dielectric sphere,

$$\alpha = \varepsilon\,\frac{\varepsilon_1-\varepsilon}{\varepsilon_1+2\varepsilon}\,a^3 \qquad (\text{sphere of radius } a, \text{ dielectric constant } \varepsilon)$$

$$\Rightarrow \quad \phi = -\frac{3}{2}\varepsilon^2\,\frac{\varepsilon_1-\varepsilon}{\varepsilon_1+2\varepsilon}\,\frac{\varepsilon_2-\varepsilon}{\varepsilon_2+2\varepsilon}\,\frac{h\nu_1\nu_2}{(\nu_1+\nu_2)}$$

so for identical molecules $(\varepsilon_1=\varepsilon_2)$ across any medium, the VdW is always attractive, but if $\varepsilon_1\neq\varepsilon_2$ and ε lies in between ε_1 & ε_2 then it is repulsive.

Approximate q.m. derivation of the London eq.:

field of a dipole $\qquad \phi(\vec{x}) = \dfrac{\vec{p}\cdot\vec{x}}{x^3}$

$$\Rightarrow \quad \vec{E}(\vec{x}) = \frac{3\hat{x}(\vec{p}\cdot\hat{x})-\vec{p}}{x^3}$$

$\vec{p}\cdot\hat{x} = p\cos\theta$; averaging: $\langle E^2\rangle = \dfrac{p^2}{x^6}\left[(3\cos\theta)^2+1-6\cos^2\theta\right]$

$$= \frac{p^2}{x^6}(3\cos^2\theta+1) = 2\frac{p^2}{x^6} \quad \text{because} \quad \langle\cos^2\theta\rangle = \frac{1}{3}$$

i.e. the averaged field from a dipole P_1

is $\quad E = \sqrt{2} \; \dfrac{P_1}{x^3}$ and the interaction potential

$$V = -\vec{P}_2 \cdot \vec{E} = -\sqrt{2} \; \frac{P_1 P_2}{x^3} \quad ; \quad \begin{aligned} \vec{P}_1 &= e\,\vec{r}_1 \\ \vec{P}_2 &= e\,\vec{r}_2 \end{aligned}$$

interaction energy in perturbation theory:

$$\Delta E = \langle 0|V|0\rangle + \sum_M \frac{|\langle 0|V|M\rangle|^2}{E_0 - E_M}$$

call $|g_i\rangle$, $|e_i\rangle$ the ground and excited state

of atom i (consider only one ~~first~~ excited state)

so a state for the system is e.g. $|g_1\rangle |e_2\rangle$ etc.

1) all diagonal elements are zero (see later)

2) $\quad \Delta E = \dfrac{2e^4}{x^6} \; \dfrac{|\langle g_1 | z_1 | e_1\rangle \langle g_2 | z_2 | e_2\rangle|^2}{\hbar\omega_1 + \hbar\omega_2}$

Polarisability of one atom:

$$V = -\vec{p}\cdot\vec{E} = -\alpha E^2 \quad \Rightarrow \quad \alpha E^2 = e^2 \bar{E}^2 \frac{|\langle g|z|e\rangle|^2}{\hbar\omega}$$

$$\Rightarrow \quad \alpha = \frac{e^2}{\hbar\omega} \, |\langle g|z|e\rangle|^2$$

$$\Rightarrow \quad \Delta E = - \frac{2 \alpha_1 \alpha_2}{x^6} \frac{\hbar \omega_1 \omega_2}{(\omega_1 + \omega_2)}$$

Why are the diagonal elements zero =

state with ang. mom. ℓ $\rightarrow$ parity $(-1)^\ell$

under parity, $z \rightarrow -z$, and given ψ_{even}

and ψ_{odd} : $z_{ee} = \langle \psi_e | z | \psi_e \rangle \rightarrow - z_{ee}$

$\Rightarrow \quad z_{ee} = 0$, also $z_{oo} = 0$
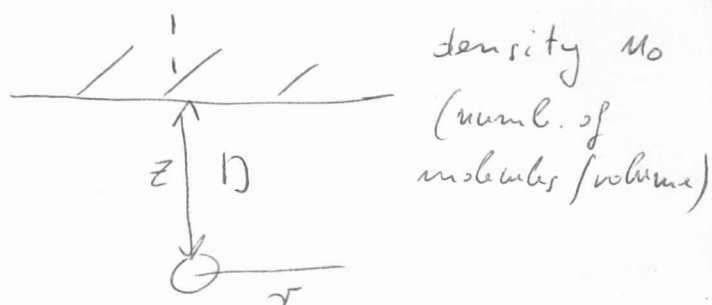
and the only allowed transitions are

e.g. $\langle \psi_e | z | \psi_o \rangle$ i.e. $\ell \rightarrow \ell \pm 1$
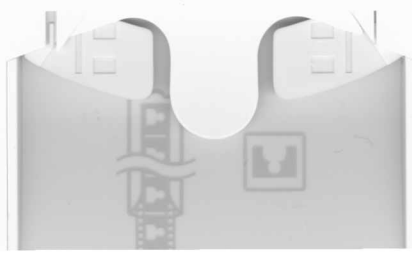
(one photon exchange) .

Interactions between macroscopic bodies (e.g. surfaces):

the power law changes ! E.g.:

molecule close to a surface

density n_0
(numb. of
molecules /volume)

$$\phi = - A \int\limits_{D}^{\infty} dz \int\limits_{0}^{\infty} 2\pi r \, dr \frac{n_0}{(z^2 + r^2)^3}$$

Change variable $\qquad z^2 + r^2 = x$

$$\Rightarrow \quad 2r\,dr = dx$$

$\rightarrow \quad$ inner integral $= M_o \pi \displaystyle\int_{z^2}^{\infty} \frac{dx}{x^3} = \frac{M_o \pi}{2} \frac{1}{z^4}$

$\Rightarrow \quad \phi = -A \frac{M_o \pi}{2} \displaystyle\int_{D}^{\infty} \frac{dz}{z^4} = -A \frac{M_o \pi}{6} \frac{1}{D^3}$

i.e. $\quad \phi \sim \dfrac{1}{D^3} \qquad$ and $\quad$ force $\sim \dfrac{1}{D^4}$.

Similarly, the interaction energy per unit area

of two surfaces separated by D is $\quad \propto \dfrac{1}{D^2}$.

Relation between the force $F(D)$ between two
curved surfaces and the interaction energy $W(D)$ (per unit area)
between planar surfaces $= \quad$ Derjaguin approx.
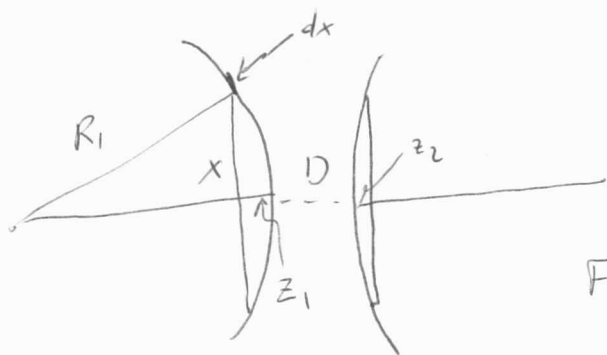
$f(z)$ force per unit area between
flat surfaces

Force between the two spheres :

$$F(D) = \int_{z=D}^{z=\infty} 2\pi x\,dx\ f(z)$$

$$x^2 = 2R_1 Z_1 = 2R_2 Z_2 \quad ; \quad Z = D + Z_1 + Z_2$$

$$= D + \frac{x^2}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$dz = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) x\, dx$$

$$\Rightarrow \quad F(D) = \int dz \quad 2\pi \, \frac{R_1 R_2}{R_1 + R_2} \, f(z)\, dz = 2\pi \, \frac{R_1 R_2}{R_1 + R_2} \, W(D)$$

$\longrightarrow$ sphere against plane : $\quad R_2 \gg R_1$

$$\Rightarrow \quad F(D) = 2\pi\, R_1\, W(D)$$